**Model Answer – Math 1-A Mid-Term Exam-I November 2015 Total mark:15**

[1] (a) A point x* is called zero to a function f(x) if

f(x*) = 0 which is the point of intersection of its curve with x–axis.

(b) A function f(x) is called even if f(–x) = f(x) and its curve is symmetric about y–axis.

------------------------------------------------------------------------------ 2 Marks

[2](a) $y = 2x^3 + 3x^2 + \sqrt[4]{2x^3 + 3x^2}$

Then $y` = 6x^2 + 6x + \frac{1}{4}(2x^3 + 3x^2)^{-\frac{3}{4}}.(6x^2 + 6x)$

(b) $y = \sin 2^x + \cos^2 x + \sec 2x$

Then $y` = \cos 2^x . 2^x \ln 2 - 2\cos x . \sin x + 2\sec 2x . \tan 2x$

(c) $y = t^2 . \log t\ ,\ x = 2^{t^3} + \tan t$

Then $y` = \frac{t^2 . \frac{1}{\ln 10} . \frac{1}{t} + 2t \log t}{2^{t^3} . \ln 2 . 3t^2 + \sec^2 t}$

(d) $y = x . 3^{x^2} + \ln(2y + 1)$

Then $y` = x . 3^{x^2} . \ln 3 . 2x + 3^{x^2} + \frac{2y`}{2y+1}$

Then $y` = \frac{2x^2 . \ln 3 . 3^{x^2} + 3^{x^2}}{1 - \frac{2}{2y+1}}$

------------------------------------------------------------------------------ 8 Marks

[3](a) $\lim_{x\to 1} \frac{\sqrt[3]{x} - 1}{1 - x^7} = \frac{0}{0} = -\frac{\frac{1}{3}}{7}(1)^{\frac{1}{3}-7} = -\frac{1}{21}$

(b) $\lim_{x\to 2} \frac{\ln(x-1)}{2 - x} = \frac{0}{0} = \lim_{x\to 2} \frac{\ln(1 + (x-2))}{-(x-2)} = -1$

(c) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{x^2 + 3x} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{\frac{\sin 2x}{x}}{x + 3} = \dfrac{2}{3}$

(d) $\lim\limits_{x\to 0} \dfrac{x^2}{2^x - 1} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{x}{\frac{2^x - 1}{x}} = \dfrac{0}{\ln 2} = 0$

(e) $\lim\limits_{x\to 0} \dfrac{\log(1 + 3x)}{\tan 2x} = \dfrac{0}{0} = \lim\limits_{x\to 0} \dfrac{\frac{\log(1 + 3x)}{x}}{\frac{\tan 2x}{x}} = \dfrac{\frac{3}{\ln 10}}{2} = \dfrac{3}{2 \ln 10}$

(f) $\lim\limits_{x\to \infty} \dfrac{x^2 + x}{2^x - 1} = \dfrac{\infty}{\infty} = \lim\limits_{x\to \infty} \dfrac{\frac{x^2}{2^x} + \frac{x}{2^x}}{1 - \frac{1}{2^x}} = \dfrac{0 + 0}{1 - 0} = 0$

----------------------------------------------------------------------------- <u>6 Marks</u>

*Dr. Mohamed Eid*

**Model Answer – Math 1-A  Mid-Term Exam-II  November 2015  Total mark:15**

[1](a) If f(x) is function, then its first derivative is given by

$$f`(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) A function f(x) is called odd if f(–x) = – f(x) and its curve is symmetric about the origin.

------------------------------------------------------------------------------------------ 2 Marks

[2](a) $y = 4x^2 + 3^x + \sqrt[3]{4x^2 + 3^x}$

Then $y` = 8x + 3^x \ln 3 + \frac{1}{3}(4x^2 + 3^x)^{-\frac{2}{3}}.(8x + 3^x \ln 3)$

(b) $y = \cos 4^x + \sin^4 x + \csc 4x$

Then $y` = -\sin 4^x . 4^x \ln 4 + 4\cos x . \sin^3 x - 4\csc 4x . \cot 4x$

(c) $y = t^3 . \log(t+2)$ ,  $x = 3^{t^2} + \cot t$

Then $y` = \frac{t^3 . \frac{1}{\ln 10} . \frac{1}{t+2} + 3t^2 \log(t+2)}{3^{t^2} . \ln 3 . 2t - \csc^2 t}$

(d) $y = x . \tan y + \log(x+3)$

Then $y` = x . sec^2 y . y` + \tan y + \frac{1}{\ln 10} . \frac{1}{x+3}$

Then $y` = \frac{\tan y + \frac{1}{\ln 10} . \frac{1}{x+3}}{1 - x . sec^2 y}$

------------------------------------------------------------------------------------------ 8 Marks

[3](a) $\lim_{x\to 1} \frac{\sqrt[3]{x} - 1}{1 - \sqrt[7]{x}} = \frac{0}{0} = -\frac{\frac{1}{3}}{\frac{1}{7}}(1)^{\frac{1}{3}-\frac{1}{7}} = -\frac{7}{3}$

$$\text{(b)}\ \lim_{x\to 0}\frac{\sin x^2}{\log(1+2x)}=\frac{0}{0}=\lim_{x\to 0}\frac{\frac{\sin x^2}{x^2}\,.\,x}{\frac{\log(1+2x)}{x}}=\frac{1.0}{\frac{2}{\ln 10}}=0$$

$$\text{(c)}\ \lim_{x\to 0}\frac{\tan 3x}{x^2-2x}=\frac{0}{0}=\lim_{x\to 0}\frac{\frac{\tan 3x}{x}}{x-2}=-\frac{3}{2}$$

$$\text{(d)}\ \lim_{x\to 0}\frac{3x}{2^x-3^x}=\frac{0}{0}=\lim_{x\to 0}\frac{3x}{3^x((2/3)^x-1)}=\frac{3}{\ln\frac{2}{3}}$$

$$\text{(e)}\ \lim_{x\to\infty}\sqrt{\frac{x^3+x}{3+4x^3}}=\frac{\infty}{\infty}=\lim_{x\to\infty}\sqrt{\frac{1+\frac{1}{x^2}}{\frac{3}{x^3}+4}}=\sqrt{\frac{1+0}{0+4}}=\frac{1}{2}$$

$$\text{(f)}\ \lim_{x\to\infty}\frac{x^2+3^x}{2^x-4^x}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{\frac{x^2}{4^x}+(\frac{3}{4})^x}{(\frac{2}{4})^x-1}=\frac{0+0}{0-1}=0$$

------------------------------------------------------------------------------ <u>6 Marks</u>

*Dr. Mohamed Eid*